Published by Institute of Physics Publishing for SISSA

Received: *April 28, 2006*
Accepted: *May 10, 2006*
Published: *May 23, 2006*

A vector supersymmetry in noncommutative U(1) gauge theory with the Slavnov term

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Abstract: We consider noncommutative U(1) gauge theory with the additional term, involving a scalar field λ, introduced by Slavnov in order to cure the infrared problem. We show that this theory, with an appropriate space-like axial gauge-fixing, exhibits a linear vector supersymmetry similar to the one present in the 2-dimensional BF model. This vector supersymmetry implies that all loop corrections are independent of the λAA-vertex and thereby explains why Slavnov found a finite model for the same gauge-fixing.

Keywords: Renormalization Regularization and Renormalons, Non-Commutative Geometry, BRST Symmetry, Topological Field Theories.

JHEP05(2006)059

*Work supported by "Fonds zur Förderung der Wissenschaftlichen Forschung" (FWF) under contract P15015-N08.

†Work supported in part by the Conselho Nacional de Desenvolvimento Científico e Tecnológico CNPq — Brazil.

Contents

1. Introduction

It is well known that noncommutative quantum field theories (NCQFT's) realized through the Weyl-Moyal $\star$ product suffer in general from the problem of UV/IR mixing [1]. This implies that one has to deal with IR singularities for vanishing external momentum. In order to get rid of the UV/IR mixing in noncommutative gauge field theories (NCGFT's) with U(1) gauge group, Slavnov [2, 3] introduced an additional term of the following form into the action:

$$\frac{1}{2}\int d^4x\,\lambda\star\theta^{\mu\nu}F_{\mu\nu}\,. \tag{1.1}$$

Here, λ represents a new *dynamical* or *"quantum"* multiplier field and the constant anti-symmetric tensor ($\theta^{\mu\nu}$) describes the noncommutativity of space-time coordinates: $[x^\mu \stackrel{\star}{,} x^\nu] = \mathrm{i}\theta^{\mu\nu}$. As a consequence of this so-called *Slavnov term*, the photon propagator becomes transversal with respect to the momentum $\tilde{k}^\mu = \theta^{\mu\nu}k_\nu$. Thereby, insertions of the (gauge independent) IR singular parts of the one-loop polarization tensor [4]

$$\Pi^{\mu\nu}_{\mathrm{IR}}(k) = \frac{2g^2}{\pi^2}\,\frac{\tilde{k}^\mu\tilde{k}^\nu}{(\tilde{k}^2)^2}\,, \tag{1.2}$$

are expected to vanish in higher-order loop calculations. For an axial gauge-fixing with $(n^\mu) = (0, 1, 0, 0)$ this result actually holds [3]. However, for a covariant gauge-fixing, new problems arise due to the fact that one has new Feynman rules including a λ-propagator, a mixed λ-photon-propagator and a corresponding vertex — see reference [5] for a detailed discussion.

In this paper, we present a new approach by identifying the Slavnov term (1.1) with a topological term. In order to preserve the unitarity of the S-matrix [6], we assume $\theta^{\mu\nu}$ to be space-like, i.e. $\theta^{0i} = 0$ in suitable space-time coordinates. Furthermore, we can choose the spatial coordinates in such a way that the only nonvanishing components of the θ-tensor are $\theta^{12} = -\theta^{21} = \theta$. Thus, the components θ^{ij} with $i, j \in \{1, 2\}$ can be written as $\theta^{ij} = \theta\epsilon^{ij}$, where ϵ^{ij} is the two-dimensional Levi-Civita symbol.[1] The Slavnov term (1.1) then reads as $\frac{\theta}{2}\int d^4x\, \lambda\star\epsilon^{ij}F_{ij}$ so that it resembles the action for a 2-dimensional BF model with Abelian gauge group [7]

$$S_{\mathrm{BF}} = \frac{1}{2}\int d^2x\, \phi\, \epsilon^{ij} F_{ij}\,. \tag{1.3}$$

The latter model represents a topological quantum field theory and it is well known that such theories exhibit remarkable ultraviolet finiteness properties at the quantum level. In particular, the 3-dimensional Chern-Simons theory and the BF models in arbitrary space-time dimension represent fully finite quantum field theories. Their perturbative finiteness relies on the existence of a linear vector supersymmetry (VSUSY for short) which is generated by a set of fermionic charges forming a Lorentz-vector [8, 9]. Together with the scalar fermionic charge of the BRST symmetry, they form a superalgebra of Wess-Zumino type, i.e. a graded algebra which closes on-shell on space-time translations. More precisely, one has the following graded commutation relations between the BRST operator s and the operator δ_μ describing VSUSY:

$$\{s, \delta_\mu\}\, \Phi = \partial_\mu \Phi \ + \ \text{contact terms}. \tag{1.4}$$

Here, Φ collectively denotes the basic fields appearing in the topological model under consideration and *contact terms* are expressions which vanish if the equations of motion are used. In this context, the axial gauge plays a special role since the topological field theories mentioned above are characterized, in this gauge, by the complete absence of radiative corrections at the loop level.

We note that the noncommutative 2-dimensional BF model is characterized, at least in the Lorentz gauge, by a VSUSY of the same form as in the commutative case [10].

The present paper is organized as follows. In sections 2, 3 and 4 we discuss the symmetries of U(1)-NCGFT with Slavnov term along the lines of topological models with an axial gauge-fixing. In section 5, we then elaborate on the consequences of these symmetries for higher-order loop calculations and in particular we show that the VSUSY infers the absence of IR divergences (which was previously pointed out by Slavnov [2, 3]).

[1] We have $\epsilon_{ij}\epsilon^{kl} = \delta_i^k\delta_j^l - \delta_i^l\delta_j^k$.

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2. Symmetries of NCGFT with Slavnov term in the axial gauge

2.1 Action

The U(1) gauge field action with Slavnov term and with an axial gauge-fixing [5] is given by

$$S = S_{\rm inv} + S_{\rm gf}\,, \quad \text{with} \begin{cases} S_{\rm inv} = \int d^4x\,(-\frac{1}{4}F_{\mu\nu}\star F^{\mu\nu} + \frac{1}{2}\lambda\star\,\theta^{\mu\nu}F_{\mu\nu}) \\ S_{\rm gf} = \int d^4x\,\,(B\star n^\mu A_\mu - \bar{c}\star n^\mu D_\mu c)\,, \end{cases} \tag{2.1}$$

where

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu - {\rm i}g\,[A_\mu \stackrel{\star}{,} A_\nu]\,, \\ D_\mu c &= \partial_\mu c - {\rm i}g\,[A_\mu \stackrel{\star}{,} c]\,. \end{aligned} \tag{2.2}$$

With $\theta^{12} = -\theta^{21} = \theta$ as the only nonvanishing components of the θ - tensor, the Slavnov term reduces to $\frac{\theta}{2}\int d^4x\,\lambda\star\epsilon^{ij}F_{ij}$, i.e. (1.3) written as an integral over 4-dimensional noncommutative space. The axial gauge-fixing vector n^μ appearing in $S_{\rm gf}$ will be chosen to lie in the plane of noncommuting coordinates, i.e. the plane (x^1, x^2), hence $n^0 = n^3 = 0$. We will see below that this allows us to find a VSUSY which is analogous to the one characterizing the 2-dimensional noncommutative BF model.

2.2 Notation

In order to distinguish the x^1, x^2-components from the other ones, we will use the following notation: Greek indices $\mu,\nu,\rho,\sigma \in \{0,1,2,3\}$ correspond to the 4-dimensional space-time, Latin indices $i,j,k,l \in \{1,2\}$ label the x^1, x^2-components and capital Latin indices $I,J,K,L \in \{0,3\}$ label the x^0, x^3-components.

For the particular choices of the axial gauge-fixing vector (n^μ) and the deformation matrix that we specified above, the action (2.1) reads as

$$S = \int d^4x\left(-\frac{1}{4}F_{\mu\nu}\star F^{\mu\nu} + \frac{\theta}{2}\lambda\star\epsilon^{ij}F_{ij} + B\star n^iA_i - \bar{c}\star n^iD_ic\right)\,. \tag{2.3}$$

It is worthwhile recalling that the star product is associative and that it has the trace property

$$\int d^4x\, f\star g = \int d^4x\, f\cdot g = \int d^4x\, g\star f\,,$$

henceforth we can perform cyclic permutations under the integral:

$$\int d^4x\, f\star g\star h = \int d^4x\, h\star f\star g = \int d^4x\, g\star h\star f\,. \tag{2.4}$$

This property will often be used in the following.

In order to simplify the notation, we will not spell out the star product symbol in the sequel: *all products between fields (or functions of fields) are understood to be star products*. Furthermore, we assume that the algebra of fields is graded by the ghost-number. Accordingly, *all commutators are considered to be graded* with respect to this degree, e.g. $\frac{1}{2}[c,c]$ stands for $\frac{1}{2}\{c \stackrel{\star}{,} c\} = c\star c$ and $[A_\mu, c]$ stands for $[A_\mu \stackrel{\star}{,} c] = A_\mu\star c - c\star A_\mu$.

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2.3 Symmetries

The action functional (2.3) is invariant under the *BRST transformations*

$$
\begin{aligned}
sA_\mu &= D_\mu c\,, \qquad & s\bar{c} &= B\,,\\
s\lambda &= -\mathrm{i}g\,[\lambda, c]\,, \qquad & sB &= 0\,,\\
sc &= \frac{\mathrm{i}g}{2}\,[c, c]\,,
\end{aligned}
\tag{2.5}
$$

which are nilpotent, i.e. $s^2\Phi = 0$ for $\Phi \in \{A_\mu, \lambda, c, \bar{c}, B\}$. The functional (2.3) is also invariant under the following *VSUSY transformations* which are labeled by a vector index $i \in \{1, 2\}$ and which only involve the x^1, x^2-components of the fields:

$$
\begin{aligned}
\delta_i A_J &= 0\,, \qquad & \delta_i c &= A_i\,,\\
\delta_i A_j &= 0\,, \qquad & \delta_i \bar{c} &= 0\,,\\
\delta_i \lambda &= \frac{\epsilon_{ij}}{\theta} n^j \bar{c}\,, \qquad & \delta_i B &= \partial_i \bar{c}\,.
\end{aligned}
\tag{2.6}
$$

The noteworthy feature of these transformations is that they relate the invariant and the gauge-fixing parts of the action (2.3). Since the operator δ_i lowers the ghost-number by one unit, it represents an antiderivation (very much like the BRST operator s which raises the ghost-number by one unit). Note that it is only the interplay of appropriate choices for $\theta^{\mu\nu}$ and n^μ which leads to the existence of the VSUSY. The crucial point is the choice of the vector n^μ lying in the plane of noncommuting coordinates.

The invariance of the action functional (2.3) under the transformations (2.6) is described by the *Ward identity*

$$
\mathcal{W}_i S \equiv \int d^4x \left(\partial_i \bar{c}\, \frac{\delta S}{\delta B} + A_i\, \frac{\delta S}{\delta c} + \frac{\epsilon_{ij}}{\theta} n^j \bar{c}\, \frac{\delta S}{\delta \lambda} \right) = 0\,. \tag{2.7}
$$

For later reference, we determine the equations of motion associated to the action (2.3). They are given by $\frac{\delta S}{\delta \Phi} = 0$ where Φ denotes a generic field. One finds that

$$
\frac{\delta S}{\delta c} = -n^i D_i \bar{c}\,, \qquad \frac{\delta S}{\delta \bar{c}} = -n^i D_i c\,, \tag{2.8a}
$$

$$
\frac{\delta S}{\delta A_i} = D_\mu F^{\mu i} + \theta \epsilon^{ij} D_j \lambda + n^i B - \mathrm{i}g n^i [\bar{c}, c]\,, \tag{2.8b}
$$

$$
\frac{\delta S}{\delta A_I} = D_\mu F^{\mu I}\,, \qquad \frac{\delta S}{\delta \lambda} = \frac{\theta}{2} \epsilon^{ij} F_{ij} = \theta F_{12}\,, \tag{2.8c}
$$

$$
\frac{\delta S}{\delta B} = n^i A_i\,. \tag{2.8d}
$$

The equation of motion for λ implements the Slavnov condition $\epsilon^{ij} F_{ij} = 0$, i.e. the vanishing of the third component of the magnetic field: $B_3 = 0$. The equation of motion for B implements an axial gauge condition $n^i A_i = 0$.

From equations (2.5) and (2.6), we can deduce the graded commutation relations of the BRST and VSUSY transformations. By using expressions (2.8), the results can be cast into the following form:

$$
[s, s]\, \Phi = [\delta_i, \delta_j]\, \Phi = 0 \qquad \text{for } \Phi \in \{A_\mu, \lambda, c, \bar{c}, B\} \tag{2.9}
$$

and

$$
\begin{aligned}
&[s,\delta_i]\,\Phi = \partial_i\Phi && \text{for } \Phi\in\{c,\bar c,B\}\,, && \text{(2.10a)}\\
&[s,\delta_i]\,A_J = \partial_i A_J - F_{iJ}\,, && && \text{(2.10b)}\\
&[s,\delta_i]\,A_j = \partial_i A_j - \frac{\epsilon_{ij}}{\theta}\frac{\delta S}{\delta\lambda}\,, && && \text{(2.10c)}\\
&[s,\delta_i]\,\lambda = \partial_i\lambda + \frac{\epsilon_{ij}}{\theta}\frac{\delta S}{\delta A_j} - \frac{1}{\theta^2}D_i\frac{\delta S}{\delta\lambda} - \frac{\epsilon_{ij}}{\theta}D_K F^{Kj}. && && \text{(2.10d)}
\end{aligned}
$$

Since contact terms appear in the graded commutators, the algebra can only close on-shell. Note that, apart from the translations, the commutators (2.10b) and (2.10d) involve some extra contributions which are not related to equations of motion. One can readily verify that these terms represent a *new symmetry* of the action (2.1) defined by the following field variations:

$$
\begin{aligned}
&\hat\delta_i A_J = -F_{iJ}\,, \qquad \hat\delta_i\lambda = -\frac{\epsilon_{ij}}{\theta}D_K F^{Kj}\,,\\
&\hat\delta_i\Phi = 0 \qquad\qquad \text{for all other fields}\,.
\end{aligned} \tag{2.11}
$$

Concerning the proof, we only note that the transformations (2.11) and the Bianchi identity imply

$$
\hat\delta_i F_{JK} = -D_i F_{JK}\,, \qquad \hat\delta_i F_{jK} = -D_i F_{jK} - D_K F_{ij}\,.
$$

Note, that the operator $\hat\delta_i$ does not change the ghost-number.

Together with the BRST transformations, the VSUSY and the translations in the (x^1,x^2)-plane,

$$
\delta_i^{(\mathrm{transl})}\Phi = \partial_i\Phi\,, \tag{2.12}
$$

this new symmetry forms an algebra which actually closes on-shell: the translations commute with all transformations and

$$
\left.\begin{aligned}
[s,s]\,\Phi = [s,\hat\delta_j]\,\Phi = 0\\
[\delta_i,\delta_j]\,\Phi = [\delta_i,\hat\delta_j]\,\Phi = 0
\end{aligned}\right\}\quad \text{for all fields } \Phi\,, \tag{2.13}
$$

$$
\begin{aligned}
&[s,\delta_i]\,\Phi = \partial_i\Phi + \hat\delta_i\Phi && \text{for } \Phi\in\{A_J,c,\bar c,B\}\,, && \text{(2.14a)}\\
&[s,\delta_i]\,A_j = \partial_i A_j + \hat\delta_i A_j - \frac{\epsilon_{ij}}{\theta}\frac{\delta S}{\delta\lambda}\,, && && \text{(2.14b)}\\
&[s,\delta_i]\,\lambda = \partial_i\lambda + \hat\delta_i\lambda + \frac{\epsilon_{ij}}{\theta}\frac{\delta S}{\delta A_j} - \frac{1}{\theta^2}D_i\frac{\delta S}{\delta\lambda}\,, && && \text{(2.14c)}
\end{aligned}
$$

and

$$
\begin{aligned}
&[\hat\delta_i,\hat\delta_j]\,A_J = \frac{\epsilon_{ij}}{\theta}D_J\frac{\delta S}{\delta\lambda}\,,\\
&[\hat\delta_i,\hat\delta_j]\,\lambda = \frac{\epsilon_{ij}}{\theta}D_J\frac{\delta S}{\delta A_J}\,,\\
&[\hat\delta_i,\hat\delta_j]\,\Phi = 0 \qquad\qquad \text{for } \Phi\in\{A_i,c,\bar c,B\}\,.
\end{aligned} \tag{2.15}
$$

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3. Generalized BRST operator

We can combine the various symmetry operators defined above into a generalized BRST operator that we denote by $\triangle$:

$$\triangle \equiv s + \xi \cdot \partial + \varepsilon^i \delta_i + \mu^i \hat{\delta}_i \qquad \text{with } \ \xi \cdot \partial \equiv \xi^i \partial_i \,. \tag{3.1}$$

Here, the constant parameters ξ^i and μ^i have ghost-number 1 and ε^i has ghost-number 2. The induced field variations read as

$$\triangle A_i = D_i c + \xi \cdot \partial A_i \,, \tag{3.2a}$$
$$\triangle A_J = D_J c + \xi \cdot \partial A_J + \mu^i F_{Ji} \,, \tag{3.2b}$$
$$\triangle \lambda = -\mathrm{i}g\,[\lambda, c] + \xi \cdot \partial \lambda + \varepsilon^i \frac{\epsilon_{ij}}{\theta} n^j \bar{c} + \mu^i \frac{\epsilon_{ij}}{\theta} D_K F^{jK} \,, \tag{3.2c}$$
$$\triangle c = \frac{\mathrm{i}g}{2}\,[c, c] + \xi \cdot \partial c + \varepsilon^i A_i \,, \tag{3.2d}$$
$$\triangle \bar{c} = B + \xi \cdot \partial \bar{c} \,, \tag{3.2e}$$
$$\triangle B = \xi \cdot \partial B + \varepsilon \cdot \partial \bar{c} \,, \tag{3.2f}$$

and imply

$$\triangle F_{iJ} = -\mathrm{i}g\,[F_{iJ}, c] + \xi \cdot \partial F_{iJ} - \mu^k D_i F_{kJ} \,.$$

Imposing that the parameters ξ^i, ε^i and μ^i transform as

$$\triangle \xi^i = \triangle \mu^i = -\varepsilon^i \,, \qquad \triangle \varepsilon^i = 0 \,, \tag{3.3}$$

we conclude that the operator (3.1) is nilpotent on-shell:

$$\triangle^2 A_i = \varepsilon^j \frac{\epsilon_{ij}}{\theta} \frac{\delta S}{\delta \lambda} \,, \tag{3.4a}$$
$$\triangle^2 A_J = \frac{\mu^i \mu^j}{2} \frac{\epsilon_{ij}}{\theta} D_J \frac{\delta S}{\delta \lambda} \,, \tag{3.4b}$$
$$\triangle^2 \lambda = \frac{\mu^i \mu^j}{2} \frac{\epsilon_{ij}}{\theta} D_J \frac{\delta S}{\delta A_J} + \varepsilon^i \frac{\epsilon_{ij}}{\theta} \frac{\delta S}{\delta A_j} - \varepsilon^i \frac{1}{\theta^2} D_i \frac{\delta S}{\delta \lambda} \,, \tag{3.4c}$$
$$\triangle^2 c = \triangle^2 \bar{c} = \triangle^2 B = 0 \,. \tag{3.4d}$$

4. Slavnov-Taylor and Ward identities

The Ward identities corresponding to the various symmetries of the action can be gathered into a Slavnov-Taylor (ST) identity expressing the invariance of an appropriate total action S_{tot} under the generalized BRST transformations (3.2), (3.3). In this respect, we introduce an external field Φ^* (i.e. an antifield in the terminology of Batalin and Vilkovisky [11]) for each field $\Phi \in \{A_\mu, \lambda, c\}$ since the latter transform non-linearly under the BRST variations — see e.g. reference [9]. We note that the external sources $A^{*\mu}$ and λ^* have ghost-number -1 whereas c^* has ghost-number -2.

4.1 ST identity

In view of the transformation laws (3.2) and (3.3), the *ST identity* reads as

$$0 = \mathcal{S}(S_{\rm tot}) \equiv \int d^4x \left\{ \sum_{\Phi\in\{A_\mu,\lambda,c\}} \frac{\delta S_{\rm tot}}{\delta\Phi^*}\frac{\delta S_{\rm tot}}{\delta\Phi} + (B+\xi\cdot\partial\bar{c})\frac{\delta S_{\rm tot}}{\delta\bar{c}} \right. \left. + (\xi\cdot\partial B+\varepsilon\cdot\partial\bar{c})\frac{\delta S_{\rm tot}}{\delta B}\right\} - \varepsilon^i\left(\frac{\partial S_{\rm tot}}{\partial\xi^i}+\frac{\partial S_{\rm tot}}{\partial\mu^i}\right). \tag{4.1}$$

This functional equation is supplemented with the *gauge-fixing condition*

$$\frac{\delta S_{\rm tot}}{\delta B} = n^i A_i\,. \tag{4.2}$$

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By differentiating the ST identity with respect to the field B, one finds

$$0 = \frac{\delta}{\delta B}\mathcal{S}(S_{\rm tot}) = \mathcal{G}S_{\rm tot} - \xi\cdot\partial\,\frac{\delta S_{\rm tot}}{\delta B}\,, \qquad \text{with}\;\; \mathcal{G}\equiv\frac{\delta}{\delta\bar{c}}+n^i\frac{\delta}{\delta A^{*i}}\,,$$

i.e., by virtue of (4.2), the so-called *ghost equation*:

$$\mathcal{G}S_{\rm tot} = \xi\cdot\partial\,(n^iA_i)\,. \tag{4.3}$$

The associated homogeneous equation $\mathcal{G}\bar{S}=0$ is solved by functionals $\bar{S}[\hat{A}^{*i},\dots]$ which depend on the variables A^{*i} and $\bar{c}$ through the shifted antifield

$$\hat{A}^{*i} \equiv A^{*i} - n^i\bar{c}\,. \tag{4.4}$$

Thus, the functional $S_{\rm tot}[A,\lambda,c,\bar{c},B\,;A^*,\lambda^*,c^*;\xi,\mu,\varepsilon]$ which solves both the ghost equation (4.3) and the gauge-fixing condition (4.2) has the form

$$S_{\rm tot} = \int d^4x\,(B+\xi\cdot\partial\bar{c})n^iA_i + \bar{S}[A,\lambda,c\,;\hat{A}^{*i},A^{*J},\lambda^*,c^*;\,\xi,\mu,\varepsilon]\,, \tag{4.5}$$

where the B-dependent term ensures the validity of condition (4.2).

By substituting expression (4.5) into the ST identity (4.1), we conclude that the latter equation is satisfied if $\bar{S}$ solves the *reduced ST identity*

$$0 = \mathcal{B}(\bar{S}) \equiv \sum_{\Phi\in\{A_\mu,\lambda,c\}}\int d^4x\,\frac{\delta\bar{S}}{\delta\hat{\Phi}^*}\frac{\delta\bar{S}}{\delta\Phi} - \varepsilon^i\left(\frac{\partial\bar{S}}{\partial\xi^i}+\frac{\partial\bar{S}}{\partial\mu^i}\right). \tag{4.6}$$

Here, $\hat{\Phi}^*$ collectively denotes all antifields, but with A^{*i} replaced by the shifted antifield (4.4). Following standard practice [9], we introduce the following notation for the external sources:

$$\rho^\mu \equiv A^{*\mu}\,,\quad \gamma\equiv\lambda^*\,,\quad \sigma\equiv c^*\,,\quad \hat{\rho}^i = \hat{A}^{*i}\,.$$

It can be checked along the usual lines (e.g. see [9]) that the *solution of the reduced ST identity* (4.6) is given by

$$
\begin{aligned}
\bar{S} = \int d^4x \, \Big\{ & -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\theta}{2} \lambda \epsilon^{ij} F_{ij} \\
& + \hat{\rho}^i \left(D_i c + \xi \cdot \partial A_i \right) + \rho^J \left(D_J c + \xi \cdot \partial A_J + \mu^i F_{Ji} \right) \\
& + \gamma \left(-\mathrm{i}g[\lambda, c] + \xi \cdot \partial \lambda + \mu^i \frac{\epsilon_{ij}}{\theta} D_K F^{jK} \right) + \sigma \left(\frac{\mathrm{i}g}{2} [c, c] + \xi \cdot \partial c + \varepsilon^i A_i \right) \\
& + \left(\frac{\mu^i \mu^j}{2} \frac{\epsilon_{ij}}{\theta} (D_J \rho^J) + \varepsilon^i \frac{\epsilon_{ij}}{\theta} \hat{\rho}^j - \varepsilon^i \frac{1}{2\theta^2} (D_i \gamma) \right) \gamma \Big\} \, . \qquad (4.7)
\end{aligned}
$$

Note that

$$\bar{S} = S_{\text{inv}} + S_{\text{antifields}} + S_{\text{quadratic}} \, ,$$

where S_{inv} is the invariant action introduced in (2.1), $S_{\text{antifields}}$ represents the linear coupling of the shifted antifields $\hat{\Phi}^*$ to the generalized BRST transformations (3.2a-d) (the $\bar{c}$-dependent term being omitted) and $S_{\text{quadratic}}$, which is quadratic in the shifted antifields, reflects the contact terms appearing in the closure relations (3.4).

4.2 The antighost and ghost equations

Differentiating the total action (4.5)–(4.7) with respect to the ghost field, one obtains

$$\frac{\delta S_{\text{tot}}}{\delta c} = D_i(\rho^i - n^i \bar{c}) + D_J \rho^J - \mathrm{i}g[\lambda, \gamma] + \mathrm{i}g[c, \sigma] + \xi \cdot \partial \sigma \, .$$

By substituting the gauge-fixing condition (4.2) in the $n^i A_i$ - dependent term on the right-hand side, we obtain the functional identity

$$\frac{\delta S_{\text{tot}}}{\delta c} + \mathrm{i}g \left[\bar{c}, \frac{\delta S_{\text{tot}}}{\delta B} \right] + n \cdot \partial \bar{c} = D_\mu \rho^\mu - \mathrm{i}g[\lambda, \gamma] + \mathrm{i}g[c, \sigma] + \xi \cdot \partial \sigma \, , \qquad (4.8)$$

which is called the *antighost equation* [9, 12]. This equation makes sense as an identity for the action functional since the right-hand side is linear in the quantum fields. Moreover it is local, i.e. not integrated, in space-time.

Similarly, differentiating the total action with respect to the antighost field, one obtains the *ghost field equation* in functional form:

$$\frac{\delta S_{\text{tot}}}{\delta \bar{c}} + \mathrm{i}g \left[c, \frac{\delta S_{\text{tot}}}{\delta B} \right] + n \cdot \partial c - \xi \cdot \partial \frac{\delta S_{\text{tot}}}{\delta B} = -\varepsilon^i \frac{\epsilon_{ij}}{\theta} n^j \gamma \, . \qquad (4.9)$$

The fact that both the ghost and the antighost field equations can be cast as such local functional identities is an expression of the ghost freedom of gauge theories quantized in an axial gauge [13].

4.3 Ward identities

The Ward identities describing the (non-)invariance of S_{tot} under the VSUSY variations δ_i, the vectorial symmetry transformations $\hat{\delta}_i$ and the translations ∂_i can be derived from the ST identity (4.1) by differentiating this identity with respect to the corresponding constant ghosts ε^i, μ^i and ξ^i, respectively.

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For instance, by differentiating (4.1) with respect to ξ^i and by taking the gauge-fixing condition (4.2) into account, we obtain the *Ward identity for translation symmetry*:

$$0 = \frac{\partial}{\partial \xi^i} \mathcal{S}(S_{\text{tot}}) = \sum_{\varphi} \int d^4x \, \partial_i \varphi \, \frac{\delta S_{\text{tot}}}{\delta \varphi} \,, \tag{4.10}$$

where $\varphi \in \{A_\mu, \lambda, c, \bar{c}, B; A^*_\mu, \lambda^*, c^*\}$.

By differentiating (4.1) with respect to ε^i, we obtain

$$\begin{aligned} 0 = \frac{\partial}{\partial \varepsilon^i} \mathcal{S}(S_{\text{tot}}) = & -\frac{\partial S_{\text{tot}}}{\partial \xi^i} - \frac{\partial S_{\text{tot}}}{\partial \mu^i} + \int d^4x \Big\{ \partial_i \bar{c} \, \frac{\delta S_{\text{tot}}}{\delta B} + (B + \xi \cdot \partial \bar{c}) \, \frac{\delta}{\delta \bar{c}} \frac{\partial S_{\text{tot}}}{\partial \varepsilon^i} \\ & + \sum_{\Phi} \left[\left(\frac{\delta}{\delta \Phi^*} \frac{\partial S_{\text{tot}}}{\partial \varepsilon^i} \right) \frac{\delta S_{\text{tot}}}{\delta \Phi} + \frac{\delta S_{\text{tot}}}{\delta \Phi^*} \left(\frac{\delta}{\delta \Phi} \frac{\partial S_{\text{tot}}}{\partial \varepsilon^i} \right) \right] \Big\} . \end{aligned} \tag{4.11}$$

From (4.5) and (4.7), we deduce that

$$\frac{\partial S_{\text{tot}}}{\partial \varepsilon^i} = \int d^4x \left\{ \sigma A_i + \frac{\epsilon_{ij}}{\theta} \hat{\rho}^j \gamma + \frac{1}{2\theta^2} \, \gamma D_i \gamma \right\} \tag{4.12a}$$

$$\frac{\partial S_{\text{tot}}}{\partial \xi^i} = \int d^4x \{ -\rho^\mu \partial_i A_\mu - \gamma \partial_i \lambda + \sigma \partial_i c \} \tag{4.12b}$$

$$\frac{\partial S_{\text{tot}}}{\partial \mu^i} = \int d^4x \Big\{ F_{iJ} \rho^J + \frac{\epsilon_{ij}}{\theta} \, (D_K F^{Kj}) \gamma + \frac{\epsilon_{ij}}{\theta} \mu^j (D_J \rho^J) \gamma \Big\} . \tag{4.12c}$$

Notice that the right-hand sides of the first two equations are linear in the quantum fields, which is not the case for the third one. Insertion of these expressions into equation (4.11) yields a *broken Ward identity for VSUSY:*

$$\mathcal{W}_i S_{\text{tot}} = \Delta_i \,. \tag{4.13}$$

Here,

$$\begin{aligned} \mathcal{W}_i S_{\text{tot}} = \int d^4x \Big\{ & \partial_i \bar{c} \, \frac{\delta S_{\text{tot}}}{\delta B} + A_i \, \frac{\delta S_{\text{tot}}}{\delta c} + \left(\frac{\epsilon_{ij}}{\theta} \, (n^j \bar{c} - \rho^j) + \frac{1}{\theta^2} D_i \gamma \right) \frac{\delta S_{\text{tot}}}{\delta \lambda} \\ & + \gamma \frac{\epsilon_{ij}}{\theta} \, \frac{\delta S_{\text{tot}}}{\delta A_j} + \left(\sigma + \frac{\mathrm{i} g}{\theta^2} \gamma \gamma \right) \frac{\delta S_{\text{tot}}}{\delta \rho^i} \Big\} \end{aligned} \tag{4.14}$$

and

$$\Delta_i = \frac{\partial S_{\text{tot}}}{\partial \xi^i} + \frac{\partial S_{\text{tot}}}{\partial \mu^i} + \int d^4x \, \frac{\epsilon_{ij}}{\theta} n^j \, (B + \xi \cdot \partial \bar{c}) \, \gamma \,. \tag{4.15}$$

More explicitly, $\Delta_i = \Delta_i \Big|_{\xi=\mu=0} + B_i[\xi, \mu]$ with

$$\begin{aligned} \Delta_i \Big|_{\xi=\mu=0} &= \int d^4x \left\{ -\rho^\mu \partial_i A_\mu + \sigma \partial_i c - \gamma \partial_i \lambda + \gamma \frac{\epsilon_{ij}}{\theta} n^j B - \rho^J F_{Ji} - \gamma \frac{\epsilon_{ij}}{\theta} D_K F^{jK} \right\} \\ B_i[\xi, \mu] &= \int d^4x \Big\{ \xi \cdot \partial \bar{c} \, \frac{\epsilon_{ij}}{\theta} n^j \gamma + \frac{\epsilon_{ij}}{\theta} \mu^j (D_J \rho^J) \gamma \Big\} . \end{aligned} \tag{4.16}$$

Several remarks concerning the results (4.13)–(4.16) are in order. First, we note that the field variations given by (4.14) extend the VSUSY transformations (2.6) by source dependent terms. It is the presence of the sources which leads to a breaking Δ_i of VSUSY — cf. the unbroken Ward identity (2.7) for the gauge-fixed action. Second, we remark that the breaking of VSUSY is *non-linear* in the quantum fields: the non-linear contributions are contained in $\Delta_i\big|_{\xi=\mu=0}$ and given by

$$-\int d^4x \left\{\rho^J F_{Ji} + \gamma \frac{\epsilon_{ij}}{\theta} D_K F^{jK}\right\} = -\int d^4x \Big\{\rho^J(\hat{\delta}_i A_J) + \gamma(\hat{\delta}_i \lambda)\Big\}\,,$$

where $\hat{\delta}_i$ are the vectorial symmetry transformations (2.11). However, these non-linear breakings (which could jeopardize a non-ambiguous definition of the theory) are contained in the derivative $\partial S_{\rm tot}/\partial\mu^i$ and are therefore functionally well defined.

Finally, we come to the Ward identity for the vectorial symmetry $\hat{\delta}_i$. By differentiating the ST identity (4.1) with respect to μ^i and using (4.12c), one finds

$$\begin{aligned}\int d^4x \Bigg\{ &- F_{iJ}\,\frac{\delta S_{\rm tot}}{\delta A_J} - \frac{\epsilon_{ij}}{\theta}\left(D_K F^{Kj} + \mu^j\, D_K\rho^K\right)\frac{\delta S_{\rm tot}}{\delta\lambda} + D_K\rho^K\,\frac{\delta S_{\rm tot}}{\delta\rho^i} \\ &+ \frac{\epsilon_{ij}}{\theta} D_K D^K\gamma\,\frac{\delta S_{\rm tot}}{\delta\rho_j} - \Big(D_i\rho^I + \frac{\epsilon_{ij}}{\theta} D^j D^I\gamma + {\rm i}g\frac{\epsilon_{ij}}{\theta}\left[F^{Ij},\gamma\right]\Big)\frac{\delta S_{\rm tot}}{\delta\rho^I} \\ &+ {\rm i}g\frac{\epsilon_{ij}}{\theta}\mu^j\left[\rho^I,\gamma\right]\frac{\delta S_{\rm tot}}{\delta\rho^I}\Bigg\} = -\int d^4x\,\frac{\epsilon_{ij}}{\theta}\,\varepsilon^j(D_K\rho^K)\gamma\,, \end{aligned} \tag{4.17}$$

i.e. we have here a breaking which is linear in the quantum fields.

5. Consequences of VSUSY

The generating functional Z^c of the connected Green functions is given by the Legendre transform[2]

$$Z^c[j_A, j_\lambda, j_B, j_c, j_{\bar{c}}] = \Gamma[A,\lambda,B,c,\bar{c}] + \int d^4x\left(j_A^\mu A_\mu + j_\lambda\lambda + j_B B + j_c c + j_{\bar{c}}\bar{c}\right)\,. \tag{5.1}$$

Thus, we have the usual relations

$$\begin{aligned} &\frac{\delta Z^c}{\delta j_A^\mu} = A_\mu\,, \quad &&\frac{\delta Z^c}{\delta j_\lambda} = \lambda\,, \quad &&\frac{\delta Z^c}{\delta j_B} = B\,, \quad &&\frac{\delta Z^c}{\delta j_c} = c\,, \quad &&\frac{\delta Z^c}{\delta j_{\bar{c}}} = \bar{c}\,, \\ &\frac{\delta\Gamma}{\delta A_\mu} = -j_A^\mu\,, &&\frac{\delta\Gamma}{\delta\lambda} = -j_\lambda\,, &&\frac{\delta\Gamma}{\delta B} = -j_B\,, &&\frac{\delta\Gamma}{\delta c} = j_c\,, &&\frac{\delta\Gamma}{\delta\bar{c}} = j_{\bar{c}}\,, \end{aligned} \tag{5.2}$$

and

$$\frac{\delta Z^c}{\delta\Phi^*} = \frac{\delta\Gamma}{\delta\Phi^*}\,, \qquad \frac{\partial Z^c}{\partial\xi^i} = \frac{\partial\Gamma}{\partial\xi^i}\,, \qquad \frac{\partial Z^c}{\partial\varepsilon^i} = \frac{\partial\Gamma}{\partial\varepsilon^i}\,, \qquad \frac{\partial Z^c}{\partial\mu^i} = \frac{\partial\Gamma}{\partial\mu^i}\,. \tag{5.3}$$

[2]In the "classical approximation", the generating functional Γ of the one-particle-irreducible Green functions is equal to the total classical action $S_{\rm tot}$. Its Legendre transform Z^c generates the connected Green functions in the tree graph approximation.

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For vanishing antifields, the *Ward identity* describing the VSUSY (4.13) becomes in terms of Z^c:

$$\mathcal{W}_i Z^c = \int d^4x \left\{ j_B\, \partial_i \frac{\delta Z^c}{\delta j_{\bar{c}}} - j_c\, \frac{\delta Z^c}{\delta j_A^i} + \frac{\epsilon_{ij}}{\theta} n^j j_\lambda\, \frac{\delta Z^c}{\delta j_{\bar{c}}} \right\} = 0\,. \tag{5.4}$$

By varying (5.4) with respect to the appropriate sources, one gets the following relations for the two-point functions (i.e. the free propagators):

$$\left.\frac{\delta^2 Z^c}{\delta j_A^i \delta j_\lambda}\right|_{j=0} = -\frac{\epsilon_{ij}}{\theta} n^j \left.\frac{\delta^2 Z^c}{\delta j_{\bar{c}} \delta j_c}\right|_{j=0}\,, \qquad \left.\frac{\delta^2 Z^c}{\delta j_A^i \delta j_A^\nu}\right|_{j=0} = 0\,. \tag{5.5}$$

The *gauge-fixing condition* (4.2) is equivalent to $n^i \frac{\delta Z^c}{\delta j_A^i} = -j_B$, from which it follows that

$$n^i \left.\frac{\delta^2 Z^c}{\delta j_B(y) \delta j_A^i(x)}\right|_{j=0} = -\delta^{(4)}(x-y)\,. \tag{5.6}$$

For vanishing antifields, the *antighost equation* (4.8) can be written as

$$-n\cdot\partial \frac{\delta Z^c}{\delta j_{\bar{c}}} - \mathrm{i}g \left[j_B, \frac{\delta Z^c}{\delta j_{\bar{c}}} \right] = j_c\,,$$

and by varying this equation with respect to j_c, one concludes that

$$n\cdot\partial \left.\frac{\delta^2 Z^c}{\delta j_c(x) \delta j_{\bar{c}}(y)}\right|_{j=0} = -\delta^{(4)}(x-y)\,. \tag{5.7}$$

Note that the same result may be obtained from the *ghost equation* (4.9) which reads in terms of Z^c (for vanishing antifields and $\xi^i = 0$):

$$-n\cdot\partial \frac{\delta Z^c}{\delta j_c} - \mathrm{i}g \left[j_B, \frac{\delta Z^c}{\delta j_c} \right] = j_{\bar{c}}\,. \tag{5.8}$$

In momentum space, the free propagators of the theory are given by

$$\mathrm{i}\Delta^{c\bar{c}}(k) = -\frac{1}{nk}\,, \qquad \mathrm{i}\Delta^{AB}_\mu(k) = \frac{\mathrm{i}k_\mu}{nk}\,, \tag{5.9a}$$

$$\mathrm{i}\Delta^{A\lambda}_\mu(k) = \frac{1}{\tilde{k}^2} \left(\tilde{k}_\mu - k_\mu \frac{n\tilde{k}}{nk} \right)\,, \tag{5.9b}$$

$$\mathrm{i}\Delta^{A}_{\mu\nu}(k) = \frac{-\mathrm{i}}{k^2} \left[g_{\mu\nu} - \frac{n_\mu k_\nu + n_\nu k_\mu}{nk} + a\frac{k_\mu k_\nu}{(nk)^2} + b(k_\mu \tilde{k}_\nu + k_\nu \tilde{k}_\mu) - \frac{\tilde{k}_\mu \tilde{k}_\nu}{\tilde{k}^2} \right]\,, \tag{5.9c}$$

with $(g_{\mu\nu}) = \mathrm{diag}\,(1,-1,-1,-1)$ and[3]

$$\tilde{k}_i \equiv \theta \epsilon_{ij} k^j\,, \qquad \tilde{k}_J \equiv 0\,,$$
$$a \equiv n^2 - \frac{(n\tilde{k})^2}{\tilde{k}^2}\,, \qquad b \equiv \frac{n\tilde{k}}{(nk)\tilde{k}^2}\,. \tag{5.10}$$

One can easily check that these propagators obey the conditions (5.5), (5.6) and (5.7).

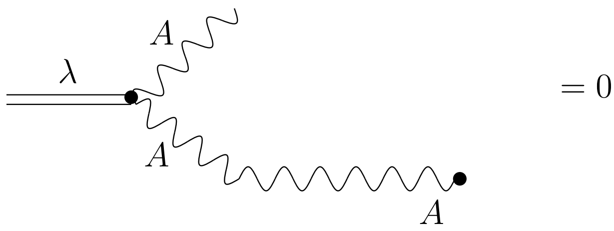

Figure 1: The λAA-vertex contracted with a photon propagator vanishes.

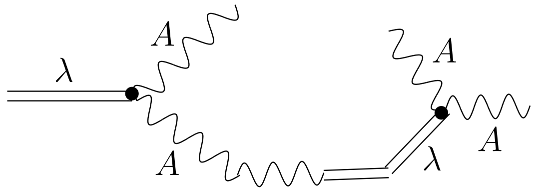

Figure 2: Building a Feynman loop graph with a λAA-vertex is impossible without a photon propagator.

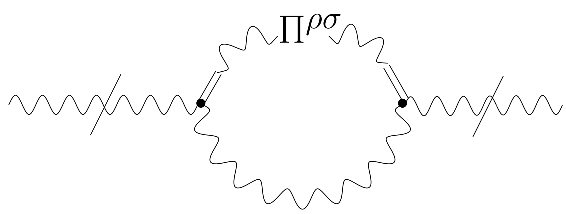

Figure 3: The "problematic" 2-loop graph vanishes in this case.

As we are now going to show, the remarkable outcome of the identities (5.5), (5.6) and (5.7) is that they are sufficient for killing all possible IR divergences in the radiative corrections. The second relation in (5.5), which states that the photon propagators $\Delta^A_{i\nu}$ vanish, has an important consequence. Indeed, since the λAA-vertex is proportional to θ^{ij}, all Feynman graphs which include a λAA-vertex contracted with an internal photon line *must cancel* (cf. figure 1). But since it is obviously impossible to construct a Feynman graph (except for a tree graph) including λAA-vertices which do not couple to internal photon propagators, *all loop corrections involving the λAA-vertex have to vanish*! Note, that a mixed photon-λ propagator contracted with a λAA-vertex leads to the necessity of another λAA-vertex, and so in order to build a closed loop, photon propagators are necessary (see figure 2). Hence, the Feynman rules involving the λ-field do not enter the loop corrections of the photon n-point function. In particular, the IR-problematic graph mentioned in our previous paper [5] and depicted in figure 3 is absent for our choice of gauge. Now that we have shown that the λ-field plays no role in the radiative corrections of the gauge field, the absence of IR-divergences follows from the line of arguments given in reference [3].

[3] We have $\tilde{k}^2 = -\theta^2(k_1^2 + k_2^2)$, $nk = -(n_1k_1 + n_2k_2)$, $n\tilde{k} = \theta(n_1k_2 - n_2k_1)$ and $\mathrm{i}\Delta^{AB}_{\mu}(x-y) = -\mathrm{i}\frac{\delta^2 Z^c}{\delta j_B(y)\delta j^\mu_A(x)}\Big|_{j=0}$.

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From these considerations, it should also become obvious that all loop corrections to the λ-propagator and the mixed λ-photon propagator vanish, leaving the tree approximation as the exact solution for this sector. Furthermore, equations (5.6) and (5.7) provide exact solutions to the AB propagator and the ghost propagator [7]. Also notice that the first of equations (5.5) is consistent with the considerations above: it gives us the exact solution for the mixed λ-photon propagator once the solution for the ghost propagator is found from (5.7).

6. Conclusion

As discussed in section 2, the U(1)-NCGFT with Slavnov term and with an appropriate axial gauge-fixing exhibits a far richer symmetry structure than initially expected. In particular, it admits a linear VSUSY which is similar to the one present in the 2-dimensional BF model, provided one chooses the deformation matrix $\theta^{\mu\nu}$ to be space-like and the axial gauge-fixing vector n^μ to lie in the plane of the noncommuting coordinates. While this VSUSY yields a superalgebra (which includes the BRST operator s and the translation generator in the noncommutative plane), it differs from the one present in the noncommutative 2-dimensional BF model by the fact that it contains an additional nonlinear vectorial symmetry (given by the transformation laws (2.11)).

As a consequence of the identities for the free propagators which follow from the VSUSY, all loop corrections become independent of the λAA-vertex. This is the reason why the theory in our particular space-like axial gauge is finite, as pointed out by Slavnov in reference [3].

Thus, the absence of IR singularities in a NCGFT can be achieved by other means than extending it to a Poincaré supersymmetric theory[4] (as was already emphasized by Slavnov [3]), namely by modifying it physically by adding the Slavnov term (which leads to the presence of VSUSY that is characteristic for a class of gauge-fixings). One may note that a supersymmetry is again responsible for the cancellation of IR singularities. But, contrary to the Poincaré supersymmetry which is physical, VSUSY is not physical, its existence following from the specific choice we have made for the gauge-fixing.[5]

Acknowledgments

Olivier Piguet would like to thank the Institut de Physique Nucléaire of the University of Lyon for a financial help, which permitted a stay during which a substantial part of this work has been done. François Gieres kindly acknowledges his stay at the University of Vitória at the final stage of the present work.

[4]The role of Poincaré supersymmetry for the cancellation of IR singularities has been extensively studied in the literature — see [14] for a review and further references.

[5]It has been shown [15] for noncommutative $\mathbf{R}^3$ that Chern-Simons models without Poincaré supersymmetry may also be free of the IR singularities, depending on the gauge-fixing choice and on the coupling with matter.

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